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Multiple photon corrections to the neutral-current Drell-Yan process

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Abstract: Precision studies of single W and Z production processes at hadron colliders require progress in the calculation of electroweak radiative corrections. To this end, higher-order QED corrections to the neutral-current Drell-Yan process, due to multiple photon radiation in Z leptonic decays, are calculated. Particular attention is paid to the effects induced by such corrections on the experimental observables which are relevant for high-precision measurements of the W-boson mass at the Tevatron Run II and the LHC. The calculation is implemented in the Monte Carlo event generator HORACE, which is available for data analysis.

Keywords: Standard Model, Electromagnetic Processes and Properties, Hadronic Colliders.

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Contents

1. Introduction **1**

2. Theoretical approach **3**

3. Numerical results and discussion **4**

3.1 Comparisons with fixed-order calculations 5

3.2 Distributions 7

3.3 Z-boson mass shifts 10

4. Conclusions and perspectives **12**

JHEP05(2005)019

1. Introduction

The Drell-Yan-like production of single W and Z bosons, with the weak boson decaying into a lepton pair, is a clean process with a large cross section at hadron colliders. It is well suited for a number of precision measurements both at the proton-antiproton ($p\bar{p}$) Fermilab Tevatron Run II and at the proton-proton (pp) CERN Large Hadron Collider (LHC) [1, 2]. While the charged-current process $pp(p\bar{p}) \to W \to l\nu_l + X$ ($l = e, \mu$) can be used for a precise determination of the W-boson mass and width, the neutral-current process $pp(p\bar{p}) \to \gamma, Z \to l^+l^- + X$ is of interest because Z data help to accurately calibrate detector parameters, such as the energy scale and resolution of the electromagnetic calorimeter, which are essential ingredients for a precise determination of the W mass [1–4]. The neutral-current process can also be used to determine the W mass from the ratio of the transverse mass distributions of the W and Z boson [5] (especially at high luminosity), as well as to extract the effective weak mixing angle from the forward-backward asymmetry [6]. Furthermore, both processes are backgrounds to many new physics searches and can also be useful to monitor the collider luminosity and measure the Parton Distribution Functions [7].

The success of precision studies in W and Z production at the Tevatron and the LHC depends on the progress in reducing theoretical uncertainties, if possible to the 1% level. To this end, QCD and electroweak corrections must be under control. Recent developments in the theory of W and Z production at hadron colliders are reviewed in ref. [8]. Order α QED corrections to the leptonic decays of W and Z bosons were calculated in ref. [9]. For the neutral-current process, which is the concern of the present study, the complete calculation of $\mathcal{O}(\alpha)$ QED corrections was presented in ref. [10], while the full set of one-loop electroweak corrections was computed in ref. [11]. To evaluate the impact of electroweak corrections in the experimental analysis, the CDF and DØ collaborations at the Tevatron

Run I made use of the fixed-order calculations of refs. [9, 12] for single W production [1] and of refs. [9 – 11] for the single Z process. However, the anticipated precision for the Drell-Yan process at the Tevatron Run II and the LHC and, in particular, for the high-precision measurement of the W mass, which depends on a precise determination of the Z parameters, requires that leading contributions from radiation of multiple soft and collinear photons are resummed through all orders of α [8, 10]. A first attempt towards the inclusion of higher-order QED corrections was the calculation of W and Z production with radiation of two additional photons in ref. [14]. Recently, higher-order corrections due to multi-photon (real and virtual) radiation in W decays at hadron colliders have been computed independently in refs. [15, 16]. Previous investigations and Monte Carlo simulations of multiple QED radiation in W/Z production in hadron collisions can be found in ref. [17]. A first step towards the inclusion of QED and QCD radiation into a single generator has been recently performed in ref. [18], by combining QCD resummation with $\mathcal{O}(\alpha)$ final-state QED corrections.

The aim of the present work is to compute higher-order final-state QED corrections to the neutral-current Drell-Yan process $pp(p\bar{p}) \to \gamma, Z \to l^+l^- + X$ and, in particular, to quantify the effects induced by such corrections on the experimental observables of interest in view of a precise determination of the W mass at hadron colliders. We restrict ourselves to consider final-state QED corrections only, because it is known from previous investigations that electroweak corrections to Drell-Yan-like processes are largely dominated by photon radiation from the final-state charged leptons [10 – 13], because of the presence of large lepton-mass logarithms arising from collinear radiation.

The present hadron collider average for the mass of the W boson M_W is $M_W = 80.454 \pm 0.059\,\text{GeV}$ [19]. The target M_W precision is of the order of 30 MeV (per channel per experiment) for the Tevatron Run II and of about 15 MeV for the LHC [1, 2], which corresponds to a high-precision determination of M_W with a relative accuracy at the 10^{-4} level. The shift in the Z mass due to multi-photon radiation was estimated at the Tevatron Run I to be less than 10 MeV by the CDF collaboration for the $Z \to \mu^+\mu^-$ decay [3] (when using the package PHOTOS of ref. [20]) and 10 MeV by the DØ collaboration for the $Z \to e^+e^-$ decay [4]. These shifts are presently treated as systematic uncertainties on the Z mass. However, in view of the expected precision for M_W in Run II and at the LHC, it will be necessary to include multiple photon corrections into account when extracting the W mass from data or when calibrating detector components using Z data. The present paper aims at completing the recent calculations of higher-order QED corrections to single W production [15, 16] to cover the process of single Z production, in order to eliminate (or largely reduce) this source of theoretical uncertainty in the experimental analysis.

The paper is organized as follows. In section 2 we describe our theoretical approach. In section 3 we present the phenomenological implications of our study, by discussing comparisons with available calculations and quantifying the effects of (higher-order) QED corrections to a number of distributions and observables of experimental interest. In section 4 we give our conclusions and perspectives.

[1] The calculation of ref. [12] refers to electroweak corrections contributing in the so-called pole approximation. Complete calculations of $\mathcal{O}(\alpha)$ electroweak corrections, beyond the pole approximation, to single W production are given in ref. [13].

JHEP05(2005)019

2. Theoretical approach

The observable cross section for the Drell-Yan process at hadron colliders with nominal squared centre of mass (c.m.) energy s is obtained by convoluting the cross section of the parton process $q\bar{q} \to \gamma, Z \to l^+l^-$, $l = e, \mu$, with the Parton Distribution Functions (PDFs) and summing over all quark flavors q. According to factorization theorems, it can be written as [2]

$$\sigma(s) = \sum_q \int d\cos\hat{\theta}\, dx_1\, dx_2\, dx_3\, dx_4 (f_{q/A}(x_1, Q^2)\, f_{\bar{q}/B}(x_2, Q^2) + (q \leftrightarrow \bar{q})) \times$$
$$\times D_{l^-}(x_3, Q^2)\, D_{l^+}(x_4, Q^2)\, \frac{d\sigma(cos\hat{\theta}, \hat{s})}{d\cos\hat{\theta}} \tag{2.1}$$

where $d\sigma(\cos\hat{\theta}, \hat{s})/d\cos\hat{\theta}$ is the parton-level differential cross section, function of the scattering angle $\hat{\theta}$ in the c.m. frame and the squared parton c.m. energy $\hat{s} = x_1 x_2 s$. The function $f_{q(\bar{q})/(A,B)}(x_i, Q^2)$ stands for the PDFs of the initial-state quarks(antiquarks) with momentum fractions x_i ($i = 1, 2$; $0 \leq x_i \leq 1$) inside the proton(antiproton), where $(A, B) = (p, \bar{p})$ for the Tevatron and $(A, B) = (p, p)$ for the LHC. The quantities $D_{l^-(l^+)}(x_i, Q^2)$ represent the QED Structure Functions (SF) for the final-state lepton(antilepton), which can be interpreted as the probability density of finding inside a parent lepton a lepton with momentum fraction x_i ($i = 3, 4$) at a virtuality scale Q^2 after photon radiation [22]. The integration is over the parton momentum fractions x_1 and x_2 and the lepton momentum fractions x_3 and x_4. This amounts to neglect the contribution of photon radiation off the initial-state quarks, as well as the contribution of initial-final-state photon interference. The inclusion in eq. (2.1) of final-state photon radiation only is meaningful from the point of view of quantum field theory because final-state QED corrections to a neutral-current $2 \to 2$ process form a gauge-invariant subset within the full set of electroweak corrections and can be therefore treated separately. Furthermore, photon radiation off (initial-state) quarks gives rise to quark mass singularities which must be reabsorbed in PDFs, in analogy to gluon emission in QCD [23] [3]. After this "renormalization" procedure, it is known from previous investigations [10 – 13] that $\mathcal{O}(\alpha)$ QED initial-state and initial-final-state interference corrections to the relevant distributions are uniform and very small (at the 0.1% level), while final-state photon corrections are seen to completely dominate and are responsible for strong modifications, at the level of several %. Consequently, initial-state and initial-final-state interference corrections contribute very little, for instance, to Z-boson mass shifts, as demonstrated in ref. [10], and can be safely neglected for the purposes of the present study.

To compute higher-order photon corrections and simulate the mechanism of multi-photon emission, we make use of the QED Parton Shower (PS) approach [25]. It consists

[2] According to eq. (2.1), the transverse motion of the Z boson is neglected. The modeling of Z transverse momentum requires careful QCD calculations, including the resummation of multiple soft-gluon radiation, as available in the Monte Carlo program RESBOS [21] used at the Tevatron.

[3] Very recently, the MRST group performed a global analysis of PDFs including QED corrections and consistently incorporated in the parameterization of PDFs the contribution of QED radiation from quarks [24].

JHEP05(2005)019

in a numerical solution of the QED Gribov-Lipatov-Altarelli-Parisi evolution equation for the lepton SF $D(x,Q^2)$ in the non-singlet channel. The solution can be cast in the form [25]

$$\begin{aligned} D(x,Q^2) = {} & \Pi(Q^2,m^2)\delta(1-x) + \\ & + \left(\frac{\alpha}{2\pi}\right)\int_{m^2}^{Q^2}\Pi(Q^2,s')\frac{ds'}{s'}\Pi(s',m^2)\int_0^{x_+}dyP(y)\delta(x-y) + \\ & + \left(\frac{\alpha}{2\pi}\right)^2\int_{m^2}^{Q^2}\Pi(Q^2,s')\frac{ds'}{s'}\int_{m^2}^{s'}\Pi(s',s'')\frac{ds''}{s''}\Pi(s'',m^2)\times \\ & \times\int_0^{x_+}dx_1\int_0^{x_+}dx_2P(x_1)P(x_2)\delta(x-x_1x_2)+\cdots \end{aligned} \tag{2.2}$$

where $P(x)$

$$P(x) = \frac{1+x^2}{1-x} - \delta(1-x)\int_0^1 dz\,\frac{1+z^2}{1-z} \tag{2.3}$$

is the regularized splitting function for the QED branching $l \to l\gamma$ and the factor Π

$$\Pi(s_1,s_2) = \exp\left[-\frac{\alpha}{2\pi}\int_{s_2}^{s_1}\frac{ds'}{s'}\int_0^{x_+}dzP(z)\right] \tag{2.4}$$

is the Sudakov form factor, representing the probability that a lepton evolves from virtuality s_2 to virtuality s_1 with no emission of photons of energy fraction greater than (an infrared regulator) $\epsilon = 1 - x_+$. Equation (2.2) allows to compute $D(x,Q^2)$ by means of a Monte Carlo algorithm which, as shown in detail in ref. [25], simulates the emission of a shower of (real and virtual) photons by a lepton and accounts for exponentiation of soft photons and resummation of collinear logarithms due to multiple hard bremsstrahlung. An $\mathcal{O}(\alpha)$ expansion of the complete PS algorithm can be worked out as well, as described in detail in ref. [25], in order to compare with fixed-order calculations. A clear advantage of the PS algorithm with respect to a strictly collinear approximation is the possibility of generating transverse momentum p_T of leptons and photons at each branching. In the present implementation of the PS algorithm, the generation of the photons' angles is performed according to the factorized part of the Z radiative decay matrix element $Z \to l^+l^-\gamma$, i.e.

$$\cos\theta_\gamma \propto -\left(\frac{p_{l^+}}{p_{l^+}\cdot k} - \frac{p_{l^-}}{p_{l^-}\cdot k}\right)^2 \tag{2.5}$$

where $p_{l^-(l^+)}$ is the lepton(antilepton) four-momentum and k is the photon four-momentum. The generation of transverse degrees of freedom at each branching allows an exclusive event generation suitable to implement experimental cuts according to a realistic event selection, as shown in the next section.

3. Numerical results and discussion

The formulation described above has been implemented in the MC event generator HORACE, developed in ref. [16] to quantify the effect of higher-order final-state QED corrections on the W-mass determination at hadron colliders. The predictions of HORACE were

JHEP05(2005)019

k_0	e		μ	
	HORACE	B&K	HORACE	B&K
0.01	41.6	41.1	22.4	22.0
0.05	24.6	24.2	13.3	12.9
0.10	17.8	17.3	9.6	9.1
0.15	13.9	13.5	7.5	7.1
0.20	11.3	10.9	6.1	5.7
0.30	7.9	7.5	4.2	3.9
0.40	5.6	5.4	3.0	2.8
0.50	4.1	3.8	2.2	2.0
0.60	2.9	2.7	1.6	1.4
0.70	2.0	1.8	1.1	0.9
0.80	1.2	1.1	0.6	0.5
0.90	0.6	0.5	0.3	0.2

Table 1: The fraction of events (in %) with a photon of energy greater than $E_\gamma^{\rm min} = k_0 E_{\rm beam}$ as predicted by the present MC program (HORACE) and the calculation of ref. [9] (B&K), at a parton c.m. energy of 90 GeV.

recently compared with those of the independent MC generator WINHAC [15] for various single-W observables of experimental interest, finding a good agreement [26]. However, in order to check the implementation in HORACE of QED corrections to the neutral-current process, we performed further numerical tests in comparison with available fixed-order calculations.

3.1 Comparisons with fixed-order calculations

First, we compared the results for the $\mathcal{O}(\alpha)$ hard-bremsstrahlung correction to the parton level cross section of ref. [9] with the corresponding $\mathcal{O}(\alpha)$ predictions of HORACE. The results of this comparison are given in table 1, which shows the fraction of events (in %) as a function of the lower cut-off k_0 ($k_0 = E_\gamma/E_{\rm beam}$, where E_γ is the photon energy and $E_{\rm beam}$ is the parton beam energy), for both electrons and muons at a parton c.m. energy of 90 GeV. In table 1 the results of Berends and Kleiss calculation of ref. [9] are denoted as B&K. The (absolute) differences between the predictions of the two calculations for the fraction of hard-photon events are at a few per mille level, as expected on the basis of the leading logarithmic approximation inherent the PS algorithm.

We also performed tuned tests between the predictions of HORACE and those of the program ZGRAD/ZGRAD2 (denoted in the following as ZGRAD2) [10, 11], in order to compare the results for hadron-level processes. The input parameters used in the comparisons with ZGRAD2, as well as in the simulations described below, are the following:

$$m_e = 0.511 \times 10^{-3}\,\text{GeV} \qquad m_\mu = 0.10565836\,\text{GeV}$$
$$\alpha^{-1} = 137.03599976 \qquad G_\mu = 1.16639 \times 10^{-5}\,\text{GeV}^{-2}$$

JHEP05(2005)019

	Tevatron Run II		LHC	
Program	e	μ	e	μ
ZGRAD2 EBA	44.11(1)	44.11(1)	557.6(1)	557.6(1)
HORACE EBA	44.07(1)	44.07(1)	557.2(1)	557.2(1)
ZGRAD2	40.84(4)	42.42(4)	510(1)	533(1)
HORACE $\mathcal{O}(\alpha)$	40.65(3)	42.22(1)	507.2(1)	530.4(1)
HORACE exponentiated	40.60(1)	42.21(1)	507.6(1)	530.5(1)

Table 2: Comparison between the present calculation (HORACE) and ZGRAD/ZGRAD2 [10, 11] (denoted as ZGRAD2) for the $pp, p\bar{p} \to \gamma, Z \to l^+l^-$, $l = e, \mu$ cross sections (in pb), at the c.m. energies of the Tevatron Run II ($\sqrt{s} = 2\,\text{TeV}$) and the LHC ($\sqrt{s} = 14\,\text{TeV}$), according to the input parameters and cuts discussed in the text.

$$M_Z = 91.1876\,\text{GeV} \qquad \sin^2\theta_W^{\text{eff}} = 0.23150 \qquad \Gamma_Z = 2.4952\,\text{GeV} \tag{3.1}$$

according to the latest PDG values [27]. In terms of these parameters, we implement in the amplitude for the parton processes $q\bar{q} \to \gamma, Z \to l^+l^-$, with $q = u, d, s, c, b$, the running of the electromagnetic coupling constant, as due to fermion loop contributions to the photon vacuum polarization [4], the running Z width in the Z propagator and effective vector and axial-vector coupling constants, in agreement with the Effective Born Approximation (EBA) described in ref. [2, 11], which yields a good description of the complete $\mathcal{O}(\alpha)$ electroweak calculation in the region around the Z pole.

The numerical results are obtained by imposing the following transverse momentum (p_T) and pseudo-rapidity (η) cuts on the final-state leptons:

$$\begin{aligned} &p_T(l^\pm) > 25\,\text{GeV} \qquad |\eta(l^\pm)| < 1.2 \qquad \text{(for the Tevatron)} \\ &p_T(l^\pm) > 25\,\text{GeV} \qquad |\eta(l^\pm)| < 2.4 \qquad \text{(for the LHC)} \end{aligned} \tag{3.2}$$

which model the acceptance cuts used by the experimental collaborations in the analysis of the Drell-Yan process. In the comparison with ZGRAD2, we impose an additional cut on the lepton-pair invariant mass $M_{l^+l^-}$, i.e. $75\,\text{GeV} \leq M_{l^+l^-} \leq 105\,\text{GeV}$, to isolate the energy region around the Z resonance. The results refer to final-state leptons in the absence of lepton identification requirements, i.e. for so-called bare leptons.

The predictions for the hadron-level processes are obtained by convoluting the parton-level matrix element with the CTEQ6 PDF set [29]. The scale Q^2 is set to be $Q^2 = \hat{s}$, $\hat{s}$ in both PDFs and lepton SFs. The c.m. energies considered are $\sqrt{s} = 2\,\text{TeV}$ for the Tevatron Run II and $\sqrt{s} = 14\,\text{TeV}$ for the LHC.

The results of the hadron-level comparison between HORACE and ZGRAD2 are shown in table 2, where the first two lines are the predictions of the programs corresponding to the cross section in the EBA, while the third and fourth lines refer to the QED corrected cross section in the presence of $\mathcal{O}(\alpha)$ final-state photon corrections. For completeness, we show in the last line also the results of HORACE when including multiple QED radiation. From

[4] The non-perturbative hadronic contributions to photon vacuum polarization is included in terms of the updated parameterization of $\Delta\alpha^{(5)}_{\text{hadrons}}$ of ref. [28].

JHEP05(2005)019

table 2, it can be seen that first-order QED corrections lower the cross section of about 10% for electrons and of about 5% for muons, both at the Tevatron and the LHC. The effect of multiple photon radiation on the integrated cross section is at the 0.1% level. The predictions of the two programs for the EBA cross sections are in very good agreement, being the relative differences below the 0.1%. Concerning the $\mathcal{O}(\alpha)$ QED corrected cross sections, the results of the two programs differ at the 0.5% level. As in the case of the parton-level comparison shown in table 1, this discrepancy can be ascribed to next-to-leading-order contributions which are included in the full $\mathcal{O}(\alpha)$ calculation implemented in ZGRAD2 and are missing in the PS algorithm of HORACE.

3.2 Distributions

Having established the physical and technical accuracy of the implementation of the neutral-current process in HORACE, we move to the presentation and discussion of the impact of higher-order corrections, with particular emphasis on the effects to the observables of interest for the measurement of the W mass, i.e. the lepton-pair invariant mass distribution $M_{l^+l^-}$ and the Z transverse mass distribution M_T^Z [1–4], which are defined as follows

$$\begin{aligned} M_{l^+l^-} &= \sqrt{(E_{l^+} + E_{l^-})^2 - (\mathbf{p}_{l^+} + \mathbf{p}_{l^-})^2} \\ M_T^Z &= \sqrt{2p_T(l^+)p_T(l^-)(1 - \cos\phi^{l^+l^-})}\,. \end{aligned} \tag{3.3}$$

In eq. (3.3) $E_{l^-(l^+)}$ and $\mathbf{p}_{l^-(l^+)}$ are the lepton(antilepton) energy and three-momentum, $p_T(l^-), p_T(l^+)$ are the transverse momenta of the lepton and antilepton and $\phi^{l^+l^-}$ is the angle between the two leptons in the transverse plane. In order to perform a more realistic phenomenological analysis and study the dependence of the QED corrections from detector effects, we implement, in addition to the cuts of eq. (3.2), the lepton identification requirements quoted in table I of ref. [12]. According to these criteria, electron and photon four-momenta are combined for small opening angles between the two particles, while muons are identified as hits in the muon chambers with an associated track consistent with a minimum ionizing particle. The four-momenta recombination for the electrons corresponds to a calorimetric particle identification, while the identification requirements for muons are close to a bare event selection. All the numerical results shown in the following refer to the cuts of eq. (3.2) for the Tevatron, at $\sqrt{s} = 2\,\text{TeV}$.

The results of our MC simulations are shown in figure 1 (invariant mass) and figure 2 (transverse mass), when considering for the final-state electrons and muons the lepton identification criteria discussed above. For the sake of comparison, we also included the results corresponding to bare electrons. The ratios shown in figure 1 and in figure 2 are defined as $\sigma_{i,\alpha}/\sigma_{i,\text{EBA}}$ and $\sigma_{i,\text{h.o.}}/\sigma_{i,\alpha}$, where $\sigma_{i,\text{EBA}}$, $\sigma_{i,\alpha}$ and $\sigma_{i,\text{h.o.}}$ are the MC predictions for the differential cross section at the EBA, $\mathcal{O}(\alpha)$ and higher-order level, for bin i, respectively. First of all, we notice that our results for the shape and size of $\mathcal{O}(\alpha)$ corrections to the invariant mass distribution are in agreement with the results of the diagrammatic calculation of ref. [10]. For both the invariant mass and transverse mass distribution and for bare leptons, the higher order corrections change sign when passing from below to above the

JHEP05(2005)019

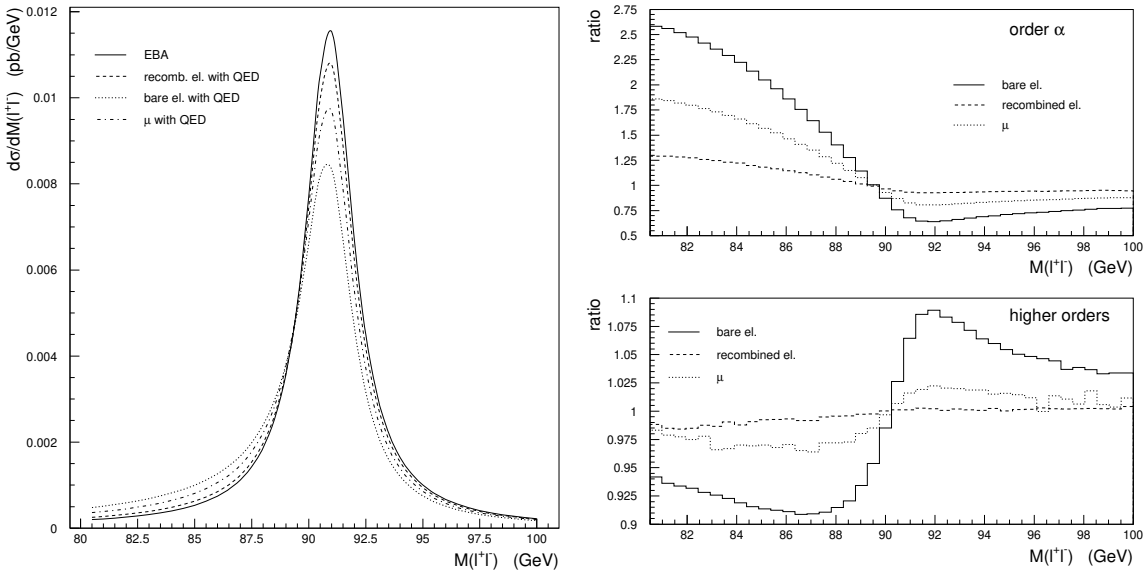

Figure 1: The invariant mass distribution $M_{l^+l^-}$ (left panel) and relative effect of $\mathcal{O}(\alpha)$ (right panel, up) and higher-order QED corrections (right panel, down) for the Z lepton decays $Z \to e^+e^-, \mu^+\mu^-$ at $\sqrt{s} = 2\,\mathrm{TeV}$, according to the lepton identification criteria discussed in the text.

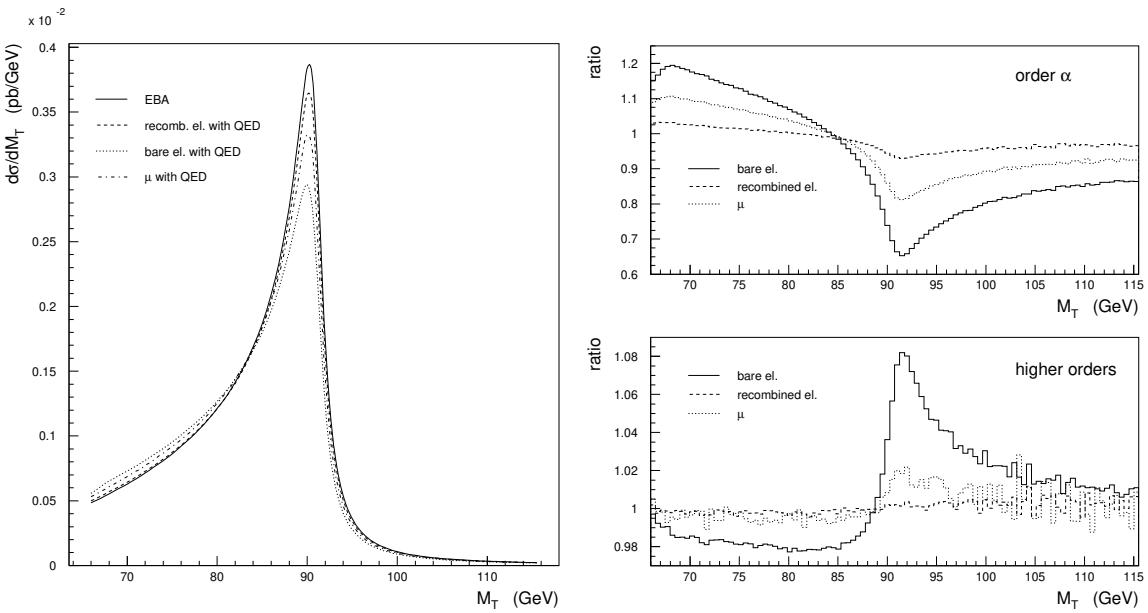

Figure 2: The Z transverse mass distribution M_T^Z (left panel) and relative effect of $\mathcal{O}(\alpha)$ (right panel, up) and higher-order QED corrections (right panel, down) for the Z lepton decays $Z \to e^+e^-, \mu^+\mu^-$ at $\sqrt{s} = 2\,\mathrm{TeV}$, according to the lepton identification criteria discussed in the text.

Z peak, and the relative correction reaches the order of 10% for bare electrons and of 1% for muons, because the collinear logarithm $\alpha \ln(\hat{s}/m_l^2)$ is larger in the electron case. After photon recombination, the higher-order correction for calorimetric electrons becomes flat and is reduced well below the 1% level, because of the disappearance of the lepton-mass

JHEP05(2005)019

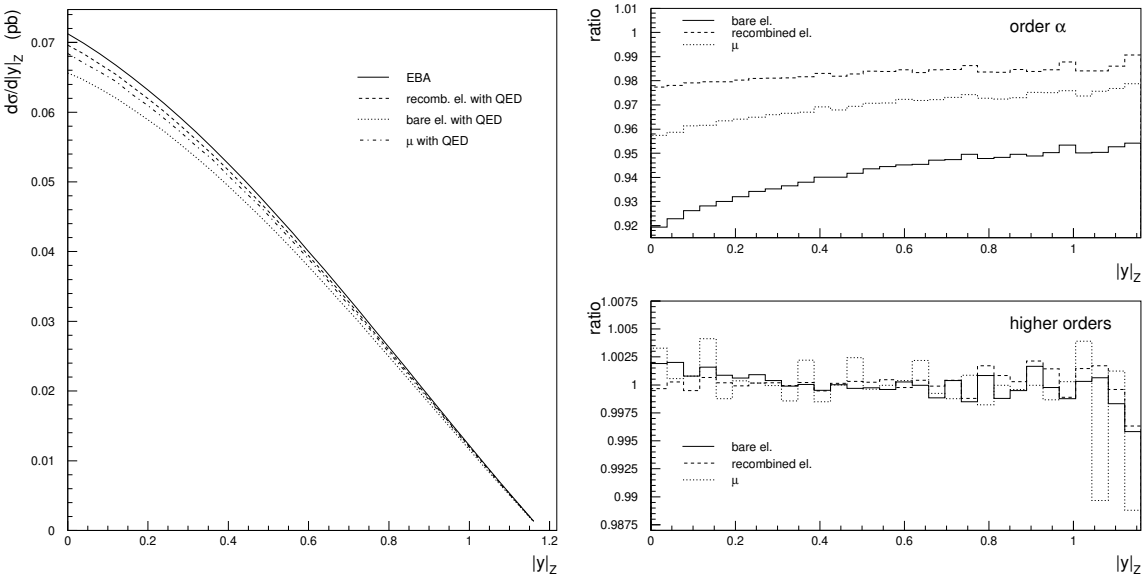

Figure 3: The Z rapidity distribution (left panel) and effect of $\mathcal{O}(\alpha)$ (right panel, up) and higher-order QED corrections (right panel, down) for the Z lepton decays $Z \to e^+e^-, \mu^+\mu^-$ at $\sqrt{s} = 2\,\text{TeV}$, according to the lepton identification criteria discussed in the text.

logarithms. We also investigated the impact of higher-order corrections on the invariant mass distribution for large values of $M_{l^+l^-}$, i.e. $M_{l^+l^-} \geq 200\,\text{GeV}$, which is important in the search for new-physics and where first-order electroweak corrections amount to several per cent [11]. We found that higher-order QED corrections are at a few % level and therefore are small in comparison with other theoretical contributions, such as Sudakov-like electroweak logarithms [11], and the expected statistical uncertainty in this invariant mass range.

In our study, we considered other observables of interest for precision electroweak measurements, such as the forward-backward asymmetry [2, 6, 10, 11], and the quantities relevant for luminosity monitoring, e.g. the lepton and Z rapidity distributions [7]. The results for the Z rapidity are shown in figure 3. We observe that $\mathcal{O}(\alpha)$ QED corrections are at the level of 3-4% for muons and of 1-2% for calorimetric electrons. They are of the same order of next-next-to-leading order (NNLO) QCD contributions recently computed in ref. [31]. The contribution of higher-order corrections is quite small, at the 0.1% level. This can be understood as follows. The luminosity observables are not strongly varying distributions and this implies that $\mathcal{O}(\alpha)$ QED corrections, and consequently higher-order corrections too, are almost flat and smaller with respect to the case of the invariant mass and transverse mass distributions. These results are in agreement with what observed for the analogous single-W observables in ref. [26]. The results for the forward-backward asymmetry A_{FB} at the Tevatron Run II are shown in figure 4 as a function of the lepton-pair invariant mass, when using for A_{FB} the definition for $p\bar{p}$ collisions given in ref. [10]. The effects of QED corrections shown in the right panel of figure 4 are defined as $A_{FB}^{\alpha} - A_{FB}^{\text{EBA}}$ and $A_{FB}^{\text{h.o.}} - A_{FB}^{\alpha}$, giving the (absolute) contribution of $\mathcal{O}(\alpha)$ and higher-order corrections, respectively. Our results for the $\mathcal{O}(\alpha)$ corrections well agree with those presented in ref. [10],

JHEP05(2005)019

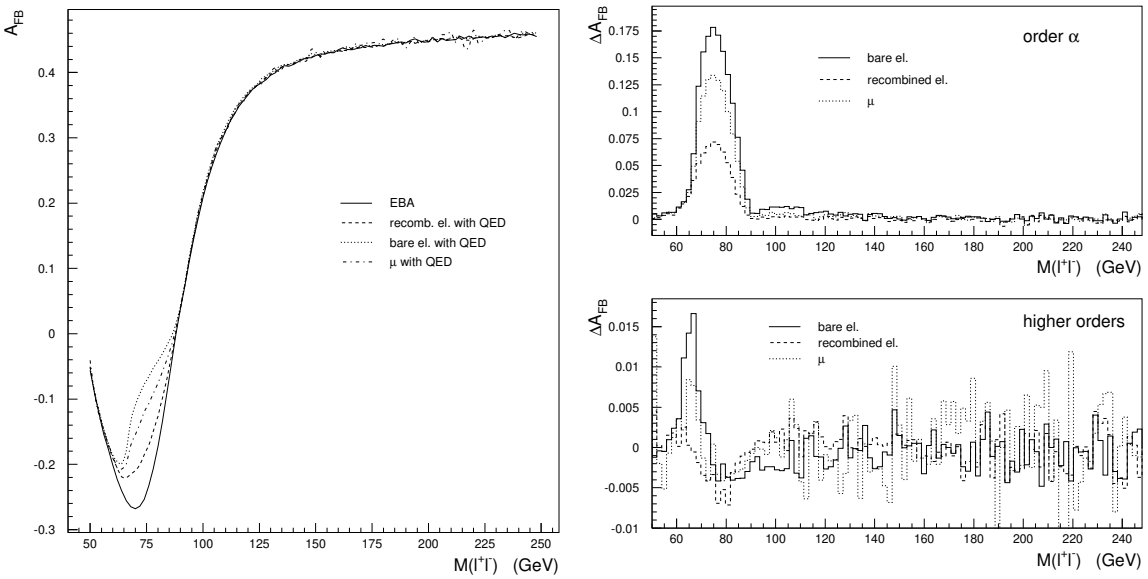

Figure 4: The forward-backward asymmetry A_{FB} (left panel) and effect of $\mathcal{O}(\alpha)$ (right panel, up) and higher-order QED corrections (right panel, down) for the Z lepton decays $Z \to e^+e^-, \mu^+\mu^-$ at $\sqrt{s} = 2\,\text{TeV}$, according to the lepton identification criteria discussed in the text.

confirming that $\mathcal{O}(\alpha)$ QED corrections are large in the region below the Z peak and small around and above it. Higher-order corrections are about a factor of ten smaller, modifying the asymmetry of about 0.01 below the peak and 0.001 above it.

3.3 Z-boson mass shifts

To complete the phenomenological analysis, we investigated the shift induced by multiple QED radiation on the Z fitted mass, which is an important parameter for calibration purposes. The strategy followed by the CDF and DØ collaborations to extract M_Z from the data is to perform a fit to the invariant mass distribution or to the Z transverse mass distribution, as a consistency check [3, 4]. In our study, we perform binned χ^2 fits to the $M_{l^+l^-}$ distribution, in analogy with the experimental procedure and following the approach already used in ref. [16] for the W mass. According to this procedure, we generate a sample of "pseudo-data" for the invariant mass distribution, calculating the distribution at the EBA level in terms of an input Z mass $M_Z^{\text{EBA,input}}$, with high numerical precision. Next, we compute the $M_{l^+l^-}$ distribution including $\mathcal{O}(\alpha)$ radiative corrections for a number of Z mass values and we calculate, for each M_Z value, the χ^2 as

$$\chi^2 = \sum_i \frac{(\sigma_{i,\alpha} - \sigma_{i,\text{EBA}})^2}{(\Delta\sigma_{i,\alpha}^2 + \Delta\sigma_{i,\text{EBA}}^2)} \tag{3.4}$$

where $\sigma_{i,\text{EBA}}$ and $\sigma_{i,\alpha}$ are the Monte Carlo predictions at the EBA and $\mathcal{O}(\alpha)$ level, respectively, for bin i and $\Delta\sigma_{i,\text{EBA}}, \Delta\sigma_{i,\alpha}$ the corresponding statistical errors due to numerical integration. From the minimum of the χ^2 distribution, we derive the fitted Z mass value $M_Z^{\alpha,\text{fitted}}$ and we quantify the mass shift due to $\mathcal{O}(\alpha)$ corrections as $\Delta M_Z^\alpha \equiv$

JHEP05(2005)019

Particle	Smearing	Lepton ID	ΔM_Z^{α} (MeV)	$\Delta M_Z^{\text{h.o.}}$ (MeV)
e	no	no	$-595\,(5)$	$135\,(1)$
e	yes	no	$-780\,(5)$	$159\,(1)$
e	no	yes	$-75\,(5)$	$5\,(1)$
e	yes	yes	$-105\,(5)$	$6\,(1)$
μ	no	no	$-270\,(5)$	$31\,(1)$
μ	yes	no	$-565\,(5)$	$49\,(1)$
μ	no	yes	$-215\,(5)$	$28\,(1)$
μ	yes	yes	$-420\,(5)$	$44\,(1)$

Table 3: The Z mass shifts due to $\mathcal{O}(\alpha)$ (ΔM_Z^{α}) and higher-order QED corrections ($\Delta M_Z^{\text{h.o.}}$), at $\sqrt{s} = 2\,\text{TeV}$, according to the different experimental conditions discussed in the text. The numbers in parentheses are statistical errors for the last digits.

$M_Z^{\text{EBA,input}} - M_Z^{\alpha,\text{fitted}}$. The shift due to higher-order corrections is derived according to the same procedure, by generating a sample of "pseudo-data" for the $M_{l^+l^-}$ distribution at $\mathcal{O}(\alpha)$ with an input Z mass value $M_Z^{\alpha,\text{input}}$ and fitting them in terms of templates of the invariant mass distribution obtained by including higher-order corrections for different M_Z values. The mass shift induced by higher-order corrections is obtained as $\Delta M_Z^{\text{h.o.}} \equiv M_Z^{\alpha,\text{input}} - M_Z^{\text{h.o.,fitted}}$, where $M_Z^{\text{h.o.,fitted}}$ is the Z mass value returned by the fit.

The results of these MC experiments are given in table 3, showing the Z mass shifts due to $\mathcal{O}(\alpha)$ (ΔM_Z^{α}) and higher-order ($\Delta M_Z^{\text{h.o.}}$) corrections for electrons and muons detected according to different combinations of experimental conditions at $\sqrt{s} = 2\,\text{TeV}$. Indeed, in addition to the shifts obtained in the presence of the lepton identification requirements discussed above and denoted as Lepton ID in table 3, we also quote the results obtained when taking into account the uncertainties in the energy and momentum measurements of the leptons in the detector. These uncertainties are simulated by means of a gaussian smearing of the particle four-momenta, using as standard deviation values the specifications relative to electrons and muons for the Run II DØ detector [30]. They are denoted as Smearing in table 3. The different combinations of smearing effects and lepton identification criteria shown in table 3 aim at emphasizing the dependence of the Z mass shift on detector specifications. In particular, it can be seen that the smearing effects tend to enhance the shifts, whereas the lepton identification requirements reduce them. When considering a specific experimental condition and comparing the $\mathcal{O}(\alpha)$ mass shift with the higher-order one, it can be noticed that higher-order corrections slightly reduce the effect due to $\mathcal{O}(\alpha)$ contributions, as expected, and that the absolute value of the Z mass shift due to multiple photon radiation is, in the most realistic case, of the order of 10% of that caused by one photon emission [5]. This agrees with the conclusions of ref. [16] for the W mass shifts due to QED effects. When using the same cuts and (simplified) detector model, we checked that

[5] The opposite sign between $\mathcal{O}(\alpha)$ and higher-order mass shifts can be understood by noticing that, in the vicinity of the Z pole, multiple photon corrections to the invariant mass distribution damp the negative effect of $\mathcal{O}(\alpha)$ corrections (see figure 1), introducing a positive contribution which makes the exponentiated QED corrected distribution closer to the EBA distribution than the $\mathcal{O}(\alpha)$ one.

JHEP05(2005)019

the above conclusions are valid for the LHC too, being the relative effect of QED corrections on the invariant mass distribution of the same order as the shifts at the Tevatron. A more realistic analysis would require a full detector simulation, as well as a modeling of the transverse motion of the Z, which are beyond the scope of the present paper.

4. Conclusions and perspectives

Drell-Yan production of Z bosons is an important process for precision measurements at hadron colliders. In particular, a precise extraction of the W mass in a hadron collider environment requires a simultaneous determination of the Z mass in lepton pair production for calibration purposes. In order to perform such precision studies at the Tevatron Run II and the LHC, radiative corrections, including the contribution of higher-order effects, must be under control. In this paper, we have presented a calculation of multiple photon final-state corrections to the neutral-current Drell-Yan process, with the aim of contributing to the progress in reducing present theoretical uncertainties. The calculation is implemented in the MC program HORACE, which can be used for data analysis. The program has been carefully tested against independent fixed-order calculations, finding good agreement.

We discussed the phenomenological implications of our calculation, concentrating on the observables which are important for a precise determination of the electroweak parameters and for luminosity monitoring. In the presence of lepton identification requirements, multiple photon corrections to the lepton-pair invariant mass and to the Z transverse mass distribution were found to modify the distributions around the Z peak by about 0.1-1%. For the forward-backward asymmetry, higher-order QED corrections modify this observable by about 0.01 below the peak and 0.001 around and above it. The quantities of interest for luminosity, such as the lepton and Z rapidity distributions, are slightly affected by multiple QED radiation, at the 0.1% level. We also investigated the shift on the Z mass, an important parameter to calibrate detector components. The precise value of the shift was found to significantly depend on detector effects (as expected), being the absolute value of the mass shift due to multiple radiation of the order of 10% of that induced by $\mathcal{O}(\alpha)$ corrections. When associated to the conclusions of refs. [15, 16, 26] for single W production, the results of the present paper indicate that the effects of multiple QED radiation are non-negligible in view of the expected precision at the Tevatron Run II and the LHC and should be carefully considered in future experimental analyses.

Concerning possible developments, it would be interesting to compare the predictions of HORACE with those of other programs at the level of differential distributions and, in particular, with the photon distributions of the package PHOTOS, which is presently used at the Tevatron to estimate the impact of two-photon radiation. A second possible perspective would be the merging of exact $\mathcal{O}(\alpha)$ electroweak corrections with the exclusive photon exponentiation of the PS algorithm, to improve the physical precision of present calculations. A further, long-term development is the combination of electroweak and QCD radiation into a single generator, to reach the required accuracy. All these developments are by now under consideration.

JHEP05(2005)019

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JHEP05(2005)019